

# Curriculum Vitae

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**Name**

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**Place and date of birth**

Milan, January 6th, 1973

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## Education and career

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- 2014 – , Associate Professor, MAT03/Geometria, Università degli Studi dell'Insubria.
- 2005 – 2014, Assistant Professor, MAT03/Geometria, Università degli Studi dell'Insubria.
- 2004, Ph.D. in Mathematics, Università degli Studi di Milano. Thesis: “Maximum and comparison principles at infinity on Riemannian manifolds”. Advisor: Prof. Marco Rigoli.
- 1999, Degree in Mathematics, Università degli Studi di Milano. Thesis: “Il teorema di Bernstein e alcune sue generalizzazioni”. Advisor: Prof. Marco Rigoli.

## Research interests

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My research activity is in the field of *Geometric Analysis*: problems of geometric nature on a Riemannian manifold concerning e.g. curvature estimates, height estimates for submanifolds, metric and topological rigidity, the structure at infinity of the space

etc., are faced using analytic tools, related especially to the qualitative study of solutions of PDEs. Very often, the Riemannian manifold is endowed with a smooth density so to include in the study a wider class of spaces of geometric interest such as the Ricci solitons and the self-shrinkers of the mean curvature flow.

**Analytic tools.** A consistent part of my research activity is focused on the development of analytic tools related to (mostly elliptic) PDEs, with the final purpose of getting information on the geometry of the underlying space. In this context, my contributions concern with: (i) Maximum and comparison principles at infinity for (possibly nonlinear) operators. Among them I would like to mention some versions, of wide use, of the celebrated Omori-Yau maximum principle. (ii) Function and potential theory (both linear and nonlinear) related e.g. to new connections between different formulations of parabolicity. (iii) The qualitative behaviour of solutions of elliptic PDEs such as uniform decay of finite-energy solutions of semilinear equations, constancy of  $L^p$  solutions involving Schrödinger operators, and gradient estimates, both  $L^p$  and uniform, for solutions of the Poisson equation. In the uniform case, the constant presents a quantitative and explicit behaviour with respect to the sectional curvature bounds of the manifold. (iv) Stochastic properties of diffusion processes from the purely PDEs viewpoint and, in particular, some new aspects of the stochastic completeness and the Feller property on a Riemannian manifold.

**Harmonic maps.** I have studied geometric and analytic aspects of ( $p$ -)harmonic functions and manifold valued maps, i.e., critical points of the ( $p$ -)energy functional. Two main lines of investigations have been followed: (i) one concerns with the vanishing and the finiteness of the space of finite energy maps. This is applied to get information on the topology at infinity of a complete manifold and on the homotopy class of a generic, but sufficiently regular, map with finite  $p$ -energy. (ii) The second line of investigation is devoted to study the solvability of the Dirichlet problem, in a given homotopy class, under different geometric assumptions on the target manifold. When the target is compact, a full understanding of the problem has been reached thus completing the program initiated by Brian White many years ago. When the target manifold is non-compact, this study involves, among other things, some new gluing constructions of Euclidean and Hyperbolic ends that are interesting in their own.

**Ricci (almost) solitons and self-shrinkers.** In the context of Ricci solitons I have worked on curvature estimates, with corresponding metric rigidity in the endpoint cases, and triviality results of the soliton structure. We have also introduced a new notion of Ricci soliton, the Ricci almost soliton, showing existence and rigidity properties both topological and metric. This new notion is somehow related to a flow introduced by Bourguignon. The contributions to the theory of self-shrinkers have gone in the direction of understanding the geometry of these submanifolds by assuming that they are confined in one of the regions bounded by hyperplanes, spheres and generalized cylinders. In particular, it is obtained the first “halfspace”-type theorem for these objects.

**Global integral inequalities and rigidity.** I have considered rigidity phenomena that arise in the presence of a Euclidean type Sobolev inequality on a complete Riemannian manifold with controlled Ricci curvature and sharp Sobolev constant. This goes in the direction of verifying a celebrated conjecture of M. Ledoux stating that a complete manifold with a sharp Euclidean Sobolev inequality is isometrically the Euclidean space. In a different direction, my interest in global integral inequalities has been focalized on a new class of  $W^{2,p}$ -inequalities, called “Calderon-Zygmund” inequalities. Their validity has

been investigated systematically e.g. under assumptions on the curvature and on the injectivity radius of the manifold. Beside natural applications to the study of solutions of the Poisson equation, this new class of integral inequalities is a promising way to investigate the geometry of a manifold subject to curvature bounds.

**Hurewicz fibrations.** Starting from the investigation on the existence of critical points of smooth maps between compact Riemannian manifolds with curvature constraints, methods from Algebraic Topology have been used to prove non-existence of certain Hurewicz fibrations between  $CW$ -complexes. The results we have obtained in this topological context are general enough to go beyond the original purpose and to include in the picture the study of almost submetries between Riemannian manifolds.

**Manifolds with boundary.** One of my most recent interests is represented by the geometry and the geometric analysis of complete manifolds with boundary. On the analytic side, I have studied potential theoretic properties related to  $L^1$ -superharmonic functions providing new localized geometric conditions for their validity, and some characterizations of parabolicity in terms of global maximum principles and Stokes type theorems. On the geometric side, there have been obtained global height estimates for constant mean curvature graphs, and Killing graphs with constant weighted mean curvature, over non-compact manifolds with boundary. In a different direction, new insights have been given on what we called “the Riemannian extension problem”. It concerns with the possibility of realizing a complete manifold with boundary as a domain inside a complete Riemannian manifold without boundary, by keeping some of the Riemannian invariants controlled. So far, the study has been focused on the Sectional and the Ricci curvatures. Positive results have been obtained for manifolds with compact convex boundaries and some systematic obstruction theory, based on topological properties of the original space, has been developed.

## Preprints

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- B. Güneysu, S. Pigola,  *$L^p$ -gradient estimates and global regularity for the Poisson equation and magnetic Schrödinger semigroups (with an appendix by Ognjen Milatovic)*. Preliminary version <https://arxiv.org/abs/1706.00591>
- S. Pigola, G. Veronelli, *The smooth Riemannian extension problem*. Preliminary version <https://arxiv.org/abs/1606.08320>

## Accepted for publication

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1. S. Pigola, G. Veronelli, *Sobolev spaces of maps and the Dirichlet problem for harmonic maps*. Comm. Contemp. Math. (to appear).
2. B. Güneysu, S. Pigola, *Quantitative  $C^1$ -estimates on manifolds*. Int. Math. Res. Notices (to appear).
3. D. Impera, J. H. de Lira, S. Pigola, A. G. Setti, *Height Estimates for Killing Graphs*. J. Geom. Anal. (to appear) DOI: 10.1007/s12220-017-9938-5.
4. D. Impera, S. Pigola, A.G. Setti, *Potential theory on manifolds with boundary and applications to controlled mean curvature graphs*. Crelle’s Journal (to appear) DOI: 10.1515/crelle-2014-0137.

1. S.L. Cacciatori, S. Pigola, *Hurewicz fibrations, almost submetries and critical points of smooth maps*. Forum Math. **29** (2017), 751–760.
2. L.F. Pessoa, S. Pigola, A.G. Setti, *Dirichlet parabolicity and  $L^1$ -Liouville property under localized geometric conditions*. J. Funct. Anal. **273** (2017), 652–693.
3. D. Impera, S. Pigola, *On the growth of supersolutions of nonlinear PDE's on exterior domains*. Nonlinear Anal. **146** (2016), 20–31.
4. S. Pigola, G. Veronelli, *On the Dirichlet problem for  $p$ -harmonic maps II: Cartan-Hadamard targets with special structure*. Proc. Amer. Math. Soc. **144** (2016), 3173–3180.
5. S. Pigola, G. Veronelli, *On the Dirichlet problem for  $p$ -harmonic maps I: compact targets* Geom. Dedicata **177** (2015), 307–322.
6. B. Güneysu, S. Pigola, *The Calderón-Zygmund inequality and Sobolev spaces on noncompact Riemannian manifolds*. Adv. Math. **281** (2015), 353–393.
7. G.P. Bessa, J.H. de Lira, A.G. Setti, *Curvature estimates for submanifolds immersed into horoballs and horocylinders*. J. Math. Anal. Appl. **431** (2015), 1000–1007.
8. S. Pigola, A.G. Setti, M. Troyanov, *The connectivity at infinity of a manifold and  $L^{q,p}$ -Sobolev inequalities*. Expositiones Math. **32** (2014), 365–383.
9. S. Pigola, A.G. Setti, *Global divergence theorems in nonlinear PDEs and Geometry*. Ensaios Matematicos 2014, Volume **26**, 1–77.
10. S. Pigola, M. Rimoldi, *Complete self-shrinkers confined into some regions of the space*. Annals Global Anal. Geom. **45** (2014), 47–65.
11. G.P. Bessa, S. Pigola, A. G. Setti, *On submanifolds of highly negatively curved spaces*. Internat. J. Math. **25** (2014) 1450055 (15 pages)
12. G.P. Bessa, S. Pigola, A.G. Setti, *On the  $L^1$ -Liouville property of stochastically incomplete manifolds*. Potential Anal. **39** (2013), 313–324.
13. G.P. Bessa, S. Pigola, A.G. Setti, *Spectral and stochastic properties of the  $f$ -Laplacian, solutions of PDEs at infinity and geometric applications*. Rev. Mat. Iberoam. **29** (2013), no. 2, 579–610.
14. S. Pigola, M. Rimoldi, *Characterizations of model manifolds by means of certain differential systems*. Canad. Math. Bull. **55** (2012), no. 3, 632–645.
15. S. Pigola, G. Veronelli, *Remarks on  $L^p$ -vanishing results in geometric analysis*. Internat. J. Math. **23** (2012), no. 1, 1250008, 18 pp.
16. S. Pigola, A.G. Setti, *The Feller property on Riemannian manifolds*. J. Funct. Anal. **262** (2012), no. 5, 2481–2515.

17. S. Pigola, M. Rigoli, M. Rimoldi, A.G. Setti, *Ricci almost solitons*. Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **10** (2011), no. 4, 757–799.
18. S. Pigola, M. Rimoldi, A.G. Setti, *Remarks on non-compact gradient Ricci solitons*. Math. Z. **268** (2011), no. 3-4, 777–790.
19. I. Holopainen, S. Pigola, G. Veronelli, *Global comparison principles for the  $p$ -Laplace operator on Riemannian manifolds*. Potential Anal. **34** (2011), 371–384.
20. S. Pigola, G. Veronelli, *Uniform decay estimates for finite-energy solutions of semi-linear elliptic inequalities and geometric applications*. Differential Geom. Appl. **29** (2011), 35–54.
21. S. Pigola, G. Veronelli, *Lower volume estimates and Sobolev inequalities*. Proc. Amer. Math. Soc. **138** (2010), 4479–4486.
22. S. Pigola, G. Veronelli, *On the homotopy class of maps with finite  $p$ -energy into non-positively curved manifolds*. Geom. Dedicata **143** (2009), 109–116.
23. S. Pigola, M. Rigoli, A.G. Setti, *Aspects of potential theory on manifolds, linear and non-linear*. Milan J. Math. **76** (2008), 229–256.
24. S. Pigola, M. Rigoli, A.G. Setti, *A finiteness theorem for the space of  $L^p$  harmonic sections*. Rev. Mat. Iberoam. **24** (2008), no. 1, 91–116.
25. S. Pigola, M. Rigoli, A.G. Setti, *Constancy of  $p$ -harmonic maps of finite  $q$ -energy into non-positively curved manifolds*. Math. Z. **258** (2008), 347–362.
26. S. Pigola, M. Rigoli, A.G. Setti, *Some characterizations of space-forms*. Trans. Amer. Math. Soc. **359** (2007), 1817–1828; **360** (2008), 3943–3944.
27. S. Pigola, M. Rigoli, A.G. Setti, *Maximum principles at infinity on Riemannian manifolds: an overview*. Workshop on Differential Geometry Mat. Contemp. **31** (2006), 81–128.
28. S. Pigola, M. Rigoli, A.G. Setti, *Some non-linear function theoretic properties of Riemannian manifolds*. Rev. Mat. Iberoam. **22** (2006), 801–831.
29. S. Pigola, M. Rigoli, A.G. Setti, *Vanishing theorems on Riemannian manifolds, and geometric applications*. J. Funct. Anal. **229** (2005), 424–461.
30. S. Pigola, M. Rigoli, A.G. Setti, *Maximum principles on Riemannian manifolds and applications*. Mem. Amer. Math. Soc. **174** (2005), no. 822.
31. S. Pigola, M. Rigoli, A.G. Setti, *Some applications of integral formulas in Riemannian geometry and PDE's*. Milan J. Math. **71** (2003), 219–281.
32. S. Pigola, M. Rigoli, A.G. Setti, *Volume growth, "a priori" estimates, and geometric applications*. Geom. Funct. Anal. **13** (2003), no. 6, 1302–1328.
33. S. Pigola, M. Rigoli, A.G. Setti, *A remark on the maximum principle and stochastic completeness*. Proc. Amer. Math. Soc. **131** (2003), 1283–1288.

34. S. Pigola, M. Rigoli, A.G. Setti, *Maximum principles and singular elliptic inequalities*. J. Funct. Anal. **193** (2002), no. 2, 224–260.
35. S. Pigola, M. Rigoli, A.G. Setti, *Some remarks on the prescribed mean curvature equation on complete manifolds*. Pacific J. Math. **206** (2002), no. 1, 195–217.

## Published books

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- S. Pigola, M. Rigoli, A.G. Setti, *Vanishing and finiteness results in geometric analysis. A generalization of the Bochner technique*. Progress in Mathematics, 266. Birkhäuser Verlag, Basel, 2008.

## Postdoc students

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- Leandro de Freitas Pessoa. Leandro received his Ph.D. in Mathematics at the Universidade Federal do Ceará. Using Brazilian funds, he has spent ten months during the a.y. 2015–2016 with a post-doc position under my direction at the Università dell’Insubria. Joint with A.G. Setti and myself, Leandro has worked on a project concerning localized geometric conditions under which one has the validity of a Liouville property for  $L^1$ -superharmonic functions.

## Ph.D. students

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- Giona Veronelli, *Some analytic and geometric aspects of the  $p$ -Laplacian on Riemannian manifolds*, a.y. 2010–2011. Currently, Giona has a position of Maître de conférences at the Université Paris 13. Personal webpage: <https://www.math.univ-paris13.fr/%7Everonelli/>
- Michele Rimoldi, *Rigidity results for Lichnerowicz-Bakry-Émery Ricci tensors*, a.y. 2011–2012. Currently, Michele has a post-doc position at the Centro Ennio de Giorgi in Pisa. Personal webpage: <http://michelerimoldi.altervista.org>

## Teaching activities

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The activities listed below have been performed at the Università degli Studi dell’Insubria.

### Teaching courses

- *Geometria I*. Focus of the course:
  - 2011–2013. Pointset topology and basic introduction to the fundamental group.
- *Geometria II*. Focus of the course:
  - 2005–2011. Covering spaces and fundamental group.
  - 2011–2012. Abstract differentiable manifolds.
  - 2012–2016. Regular surfaces in  $\mathbb{R}^3$ .

- *Geometria III*. Focus of the course:
  - 2005–2006. Local theory of smooth curves in the space and global theory of smooth curves in the plane.
  - 2006–2017. Abstract differentiable manifolds.
- *Metodi Geometrici in Analisi Numerica*. Focus of the course:
  - 2007–2009. Morse Theory.
- *Istituzioni di Geometria Superiore (Topics in Advanced Geometry)*. Focus of the course:
  - 2014–2015. Abstract differentiable manifolds.
- *Geometria Superiore (Advanced Geometry)*. Focus of the course:
  - 2013–2014, 2015–2016. Riemannian Geometry.

### Complementary teaching activities

- *Precorso di Matematica*. 2010–2011, 2012–2013, 2014–2015.
- *Stage Estivo di Matematica e Applicazioni*. 2006–2013, 2015–2016. Topics: R.S.A. cryptography.

### Master students

The following is a list of master theses written under my supervision.

1. Luigi Caspani, *Omologia simpliciale  $L^p$  per complessi simpliciali non-compatti*, a.y. 2005–2006
2. Giona Veronelli, *Ipersuperfici a curvatura totale finita nello spazio iperbolico: stime di curvatura e struttura all'infinito*, a.y. 2006–2007.
3. Michele Rimoldi, *Caratterizzazione di modelli mediante equazioni differenziali*, a.y. 2007–2008.
4. Davide Calza, *Teoremi di rigidità per disuguaglianze ottimali di tipo Poincaré-Sobolev su varietà Riemanniane*, a.y. 2009–2010.
5. Chiara Piatti, *Gruppo fondamentale e curvatura di Ricci: la congettura di Milnor*, a.y. 2010–2011.
6. Lorenzo Favilli, *Aspetti topologici e differenziali dei bracci meccanici*, a.y. 2011–2012.
7. Silvia Ballabio, *Complessità asintotica delle immersioni isometriche nello spazio Euclideo di varietà di Hadamard*, a.y. 2014–2015.
8. Alberto Roncoroni, *Domain rigidity for some overdetermined elliptic boundary value problems*, a.y. 2015–2016.
9. Nicolò De Ponti, *Concentrazione della misura e applicazioni geometriche*, a.y. 2015–2016.

- *A Geometry Day in Como.* It is a one-day conference devoted to various aspects of differential geometry, global analysis and geometric analysis both on smooth and on singular spaces. The “Geometry Day” takes place regularly since 2013, during the first half of January, at the *Dipartimento di Scienza e Alta Tecnologia* in Como. It consists of 4 or 5 talks of 1h each. More information can be found at the link: <http://www.dfm.uninsubria.it/pigola/geometryday.html>.
- *Summer School: Geometric Analysis on Riemannian and singular metric measure spaces.* The aim of the school is to introduce Ph.D. students and young researchers to cutting edge ideas and recent techniques in the geometric analysis of both smooth and singular metric measure spaces. Courses are conducted at the Ph.D. level by world leading experts. The school consists of 5 main courses of 3 - 5 hours each. Moreover, students have the opportunity to give 30 min talks presenting their research on themes related to the main topics of the courses. Informal discussions are strongly encouraged, and specific time slots will be reserved for this important part of the school. More information on the past two editions can be found at the link: <http://arms.lakecomoschool.org>.

## Talks and mini-courses

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- *Some a-priori estimates for the Poisson equation on complete manifolds.* Roma, La Sapienza. April 19th, 2017.
- *Riemannian manifolds with boundary and smooth domains.* Geometry Seminars, Pavia, November 3, 2016.
- *Riemannian manifolds with boundary and smooth domains.* Geometric Analysis in Samothrace, a tribute to Gérard Besson. May 30-June 4, 2016.
- *Extending Riemannian manifolds past their boundaries.* A Differential Geometry Day in memory of Sergio Console. Torino, 13 May 2016.
- *The Dirichlet problem for harmonic maps into convex supporting balls.* Workshop 2015, Varietà reali e complesse: geometria, topologia e analisi armonica. SNS di Pisa, 5-7 March, 2015.
- *Geometric aspects of recurrence, non-explosion and Feller property of a Riemannian manifold.* XVIII Escola de Geometria Diferencial, Brasilia, July 28th-1st August 2014.
- *Some geometric aspects of parabolicity, stochastic completeness and Feller property.* Topics in Geometric Analysis, Potsdam, June 19th 2014.
- *Height estimates for graphs of constant mean curvature.* A meeting with Louis Nirenberg, Varese 10-13 June 2014.
- *A survey of stochastic properties of Riemannian manifolds and their geometric applications.* VII Workshop on Geometric Analysis, Fortaleza 17-21 February 2014.



- *Geometric aspects of the potential theory on Riemannian manifolds*. Université Paris 13, December 10th, 2013.
- *Alcuni aspetti geometrici della teoria del potenziale su varietà Riemanniane*. Applied Mathematics Seminar, Pavia, April 23th, 2013.
- *Stochastic properties of manifolds: Liouville-type aspects*. Maceiò, February 2012. II workshop of Differential Geometry.
- *Global divergence theorems in nonlinear PDEs and Geometry*. Fortaleza, January 2012. Mini-course for the Summer School in Differential Geometry.
- *Geometric aspects of the  $p$ -Laplacian on complete manifolds*. Grenoble, September 2011. Workshop Geometric analysis II Institut Fourier - Brazil.
- *$p$ -Laplacian and topology of manifolds*. Santiago de Compostela, December 2010. Conference in Geometry and Global Analysis.
- *Some analytic and geometric aspects of the  $p$ -Laplacian on Riemannian manifolds*. Bardonecchia, June 2009. Convegno Nazionale di Analisi Armonica.
- *Some vanishing and finiteness results on complete manifolds: a generalization of the Bochner technique*. Caramanico Terme, May 2007. Convegno Nazionale di Analisi Armonica.
- *Some vanishing and finiteness results on complete manifolds: a generalization of the Bochner technique*. Università degli Studi di Roma "La Sapienza", January 2007. Seminario di Topologia Algebrica e Differenziale.
- *Some topics in the theory of harmonic functions on complete Riemannian manifolds*. Università degli Studi di Milano Bicocca, July 2006.

## Professional service

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- Referee for: *Note di Matematica*, *American Journal of Mathematics*, *Manuscripta Mathematica*, *Journal of Differential Equations*, *Journal of Mathematical Analysis and Applications*, *Journal of Geometry and Physics*, *Communications in Contemporary Mathematics*, *Monatshefte für Mathematik*, *Geometriae Dedicata*, *Journal of Geometric Analysis*, *Differential Geometry and its Applications*, *Publicationes Mathematicae Debrecen*, *Annali della Scuola Normale Superiore di Pisa*, *Potential Analysis*, *Journal of Differential Equations*, *Annals of Global Analysis and Geometry*, *Pacific Journal of Mathematics*, *Annali di Matematica Pura e Applicata*, *Journal of Nonlinear Analysis*, *Funkcialaj Ekvacioj*, *Communications in Analysis and Geometry*, *Results in Mathematics*.