

# Notions of ‘Point’ in Point-Free Topology

J.L. Wrigley

In this presentation we give an introduction to the field of ‘point-free topology’ with a focus on recovering points. In the later stages, we make reference to forthcoming work in [1] and [4] on internal locales of Grothendieck topoi.

As extolled in [2], topological spaces and continuous maps are generalised by locales and locale morphisms which capture the algebraic aspects of topology without reference to points. We will recall the definition of a locale and locale morphism and discuss the notion of ‘point’ of a locale. We construct the topological space  $\text{Pt}(L)$  of points of a locale  $L$ , the closest approximation of  $L$  by a topological space. In the process, we arrive at the well-known adjunction

$$\mathbf{Top} \begin{array}{c} \xrightarrow{\Omega} \\ \perp \\ \xleftarrow{\text{Pt}} \end{array} \mathbf{Loc}$$

relating topological spaces and locales.

Finally, we sketch the extension of the notion of locale found in topos theory: *internal locales*, first characterised over a Cartesian base in [3]. We also elucidate the relationship between the notion ‘internal point’ and ‘external point’.

## References

- [1] O. CARMELLO, *Fibred Sites and Existential Toposes*, (Forthcoming), 2022.
- [2] P.T. JOHNSTONE, *The Point of Pointless Topology*, Bull. Amer. Math. Soc., vol. 8, no. 1, pp. 3-17, 1981.
- [3] A. JOYAL & M. TIERNEY, *An Extension of the Galois Theory of Grothendieck*, Mem. Amer. Math. Soc., vol. 51, 1984.
- [4] J.L. WRIGLEY, *Externalised Internal Locales*, (Forthcoming), 2022.