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EINSTEIN EQUATIONS: PHYSICAL AND MATHEMATICAL ASPECTS OF GENERAL RELATIVITY

SHORT TALKS

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Spatial measurement in curved spacetime

Abstract

I examine the observer dependence of length measurement in general relativity. Given a spatial vector $\boldsymbol{\xi}$ representing an unstressed ruler, and a coordinate Φ which serves as an extrinsic reference, the proper length element is

$$dL = \frac{1}{d\Phi(\boldsymbol{\xi})} d\Phi.$$

This situates extended objects, relative to Φ, in their rest frame, generalising length-contraction from special relativity. Given an observer $\mathbf{u}$, the ruler direction which maximises $dL/d\Phi$ is shown to have measurement

$$dL_{\Phi\text{-max}} = \frac{1}{\sqrt{g^{\Phi\Phi} + (u^\Phi)^2}} d\Phi.$$

As a specific example, consider radial motion in Schwarzschild spacetime, parametrised by the Killing energy per mass e. Then the radial proper length is:

$$dL = \frac{1}{|e|} dr$$

(Gautreau & Hoffmann 1978), which remains valid inside the horizon, and reduces to the familiar quantity $(1 - 2M/r)^{-1/2} dr$ in the case of static observers. Only observers with $dr/d\tau = 0$ can possibly orient their rulers to achieve the usual measurement.

The formalism here is remarkably "post-mature", and is independent of the excellent work by de Felice & Bini (2010), as well as spacetime splitting formalism (Jantzen+ 2013) to which it is related.